

Beta-gamma directional correlation in Rb^{86}

By J. H. HAMILTON,¹ B.-G. PETTERSSON, and J. M. HOLLANDER²

ABSTRACT

The directional correlation of the once-forbidden, non-unique 700 keV beta group with the 1.08 MeV gamma-ray in the decay of Rb^{86} has been measured as a function of beta energy from 100 keV to 600 keV. A magnetic lens spectrometer has been used for the energy selection of the beta particles. The maximum anisotropy measured at 600 keV is $A = 0.40$, which is much higher than the value reported earlier by Stevenson and Deutsch. An analysis of the beta transition is made in terms of the "modified B_{ij} approximation".

Introduction

The determination of the nuclear matrix elements in once-forbidden beta decay has received much attention theoretically [1 a-d, 2]. The Rb^{86} once-forbidden beta group has received special attention as an example which offers the possibility to gain information about the nuclear matrix elements in 2^- to 2^+ beta decays [1 d]. As is noted in the discussion below, the determination of these matrix elements can be made only if several observables are known. One especially useful result in these determinations is the beta-gamma directional correlation and, particularly, the beta-energy dependence of this correlation.

The Rb^{86} beta-gamma directional correlation of the 2^- to 2^+ group (see Fig. 1) has been measured earlier by Stevenson and Deutsch [3]. Because of the rather thick sources used in their measurements, their results are rather uncertain at low beta energies. Furthermore, they found that the anisotropy decreased at around 600 keV, which is inconsistent with the results obtained from the measurements of other observables.

Because of the importance of the beta-gamma directional correlation in the determination of the nuclear matrix elements in the Rb^{86} decay, we have remeasured the correlation under improved conditions. Special emphasis was given to the beta-energy dependence of the correlation. A preliminary report of this work has appeared [4]. Our new result is a much larger anisotropy than that reported by Stevenson and Deutsch [3].

In addition to the beta-gamma directional correlation, the shape of the 2^- to 2^+ beta group is known [5], as well as the beta-circularly polarized gamma correlation [6] and the branching ratio for the 2^- to 2^+ and 2^- to 0^+ beta groups [7]. In this paper, an attempt is made to obtain a consistent agreement with theory for these observables.

¹ National Science Foundation Post-doctoral Fellow on leave from Vanderbilt University.

² Guggenheim Fellow on leave from University of California Lawrence Radiation Laboratory.

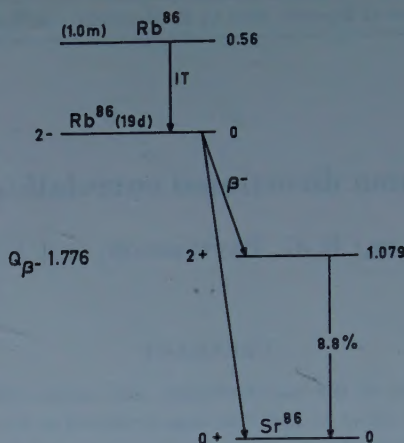


Fig. 1. Decay scheme as given in Strominger, Hollander and Seaborg [10].

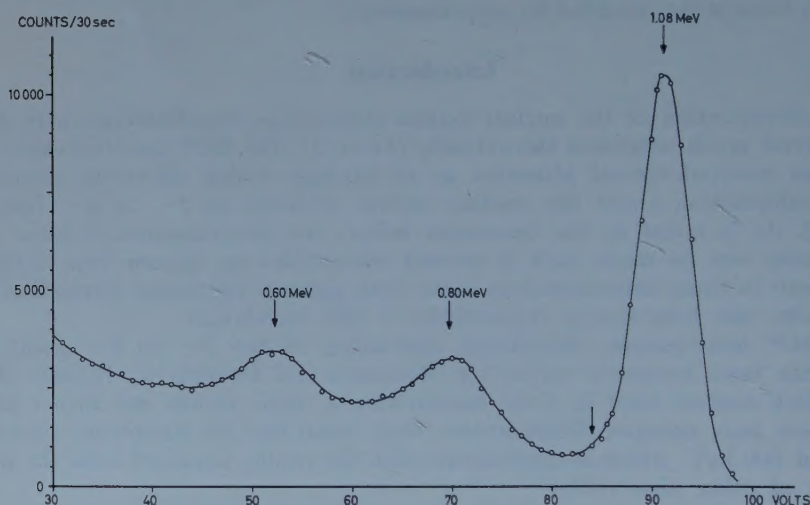


Fig. 2. Gamma-ray spectrum as measured with a $2\frac{1}{2}'' \times 2''$ NaI(Tl) crystal. The strong peak is from Rb^{86} and the lower energy ones are from Cs^{134} contamination. The arrow indicates the discriminator setting.

Source and apparatus

The rubidium activity was produced by means of the $\text{Rb}^{85}(n, \gamma)\text{Rb}^{86}$ reaction in DIDO (Harwell) at a flux of 10^{14} neutrons/cm²/sec, which gave a specific activity of 6 Curies/gram. The spectrometer source was made by evaporating the rubidium solution onto a 200 $\mu\text{g}/\text{cm}^2$ aluminum backing. The activity was found to be confined within and uniformly spread over a circular spot of 5 mm diameter. Estimation of the source strength from the photopeak counting rate of the 1.08 MeV gamma ray implied an average source thickness of 80 $\mu\text{g}/\text{cm}^2$.

A magnetic lens spectrometer was used for the energy selection of the beta particles. The momentum resolution of the spectrometer was set at 6 per cent. For detection of the gamma rays, a $2\frac{1}{2}'' \times 2''$ NaI(Tl) crystal coupled to a Du-Mond 6363 photomultiplier tube was utilized. A detailed description of the instrument is given by Gerholm and Pettersson [8].

Measurements and results

The measured photon spectrum revealed that the irradiated rubidium sample contained about 0.1 per cent Cs^{133} . The relatively high cross-section for the reaction $\text{Cs}^{133}(n, \gamma)\text{Cs}^{134}$ and the small abundance of the beta-decay branch in Rb^{86} of interest (see Fig. 1) makes even such a small contamination disturbing, as beta-gamma coincidences from the decay of Cs^{134} might influence the measured correlation. To reduce such coincidence contributions, only the photopeak of the 1.08 MeV gamma ray in Rb^{86} was accepted in the gamma channel (see Fig. 2). In this way the only coincidences measured are those of gamma rays with energy higher than about 1 MeV which are coincident with beta particles with energies between 100 keV and 600 keV. From the known decay scheme [9] of Cs^{134} one can deduce that at all the beta energies measured in this work, the contribution from Cs^{134} coincidences was less than 2 parts in 10^3 of the true Rb^{86} coincidence counting rate. The contribution to the correlation from these coincidences is negligible.

Coincidences were collected at the three angles 90° , 180° , and 270° after the source had been so centered that the single gamma counting rates at these angles differed by less than 0.5 per cent. This correction factor for the effective solid angles at the different angular positions was checked each day to an accuracy of 0.1 per cent and was found to be constant during the entire measuring period (40 days). The beta counting rate was found to be independent of the gamma counter position to within 0.1 per cent, over the whole beta spectrum. The angular position of the gamma counter was changed every 1000 seconds to avoid effects on the correlation from possible electronic drifts.

The resolving time of the coincidence unit was set at $2\tau = 3.5 \times 10^{-8}$ seconds, and from frequent checks this was found to be constant to within 3 per cent. The ratio of true to accidental coincidences varied from 7 at the highest beta energy to 50 at the lowest beta energy measured.

The beta-gamma correlation was measured at eleven beta energies between 100 keV and 600 keV. At each point, 60,000 to 100,000 coincidences were collected.

The anisotropy A was calculated from the formula

$$A = \frac{N_c(180^\circ) \cdot N_\gamma(90^\circ)}{N_c(90^\circ) \cdot N_\gamma(180^\circ)} - 1$$

where $N(90^\circ)$ stands for the average of the 90° and 270° data.

The expression $\frac{N_c(90^\circ) \cdot N_\gamma(270^\circ)}{N_\gamma(90^\circ) \cdot N_c(270^\circ)}$ was within statistics equal to unity for all measured beta energies.

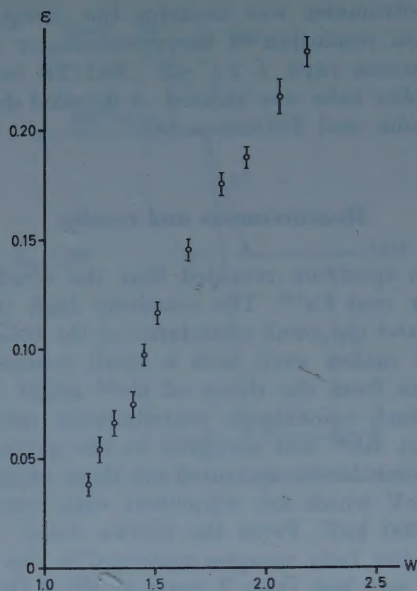


Fig. 3. The beta-gamma directional correlation coefficient ε as a function of energy.

The correlation function was assumed to be of the form

$$N(\theta) = 1 + \varepsilon P_2(\cos \theta)$$

from which it follows that

$$\varepsilon = \frac{2A}{3+A}.$$

A plot of ε (corrected as explained below) as a function of the beta energy is given in Fig. 3. The error bars correspond to one standard deviation in the data. A strong positive correlation is observed, with maximum anisotropy $A = 0.40$.

A small correction for source scattering has been applied. This correction is less than 3 per cent even at 100 keV. The measured coefficients have further been corrected for the finite solid angles subtended by the detectors. These corrections are $A_\gamma = 0.95$ and $A_\beta = 0.91$ in the equation

$$\varepsilon(\text{corrected}) = \frac{1}{A_\gamma} \cdot \frac{1}{A_\beta} \cdot \varepsilon(\text{measured}).$$

Discussion

The maximum measured anisotropy, $A = 0.40$, is about a factor two higher than the anisotropy found by Stevenson and Deutsch [3]. A possible explanation for this discrepancy is that their correlation was affected by the presence of Cs¹³⁴ coincidences. Interestingly, we found that the results of Stevenson and Deutsch

could be well reproduced with our source when all gamma rays with energy above 300 keV were accepted in the gamma channel.

The following is a brief theoretical analysis of our results. We shall follow the notation used by Kotani and Ross in their recent papers [1].

In a $2- \rightarrow 2+$ beta transition, matrix elements of rank 0, 1, and 2 can be expected to contribute to the beta transition probability. In the Kotani and Ross notation, these are written as

$$\begin{aligned} \eta w &= C_A \int \underline{\sigma} r, & \eta \xi' v &= C_A \int i \gamma_5 & \text{for } \lambda=0, \\ \eta u &= C_A \int i \underline{\sigma} \times r, & \eta \xi' y &= -C_v \int i \underline{\alpha}, & \eta x &= -C_v \int r & \text{for } \lambda=1, \\ \eta z &= C_A \int B_{ij} & & & \text{for } \lambda=2. \end{aligned}$$

The nuclear parameters, u, v, w, x, y , and z are the ratios of the various matrix elements compared to a standard matrix element η . $|\eta|^2$ will be a common factor in the transition probability which can be arbitrarily defined. It is convenient to take

$$|\eta|^2 = \left| C_A \int B_{ij} \right|^2$$

from which $z=1$. Instead of the relativistic matrix elements contained in v and y , the parameters

$$V = \xi' v + \xi w \quad \text{for } \lambda=0$$

$$\text{and} \quad Y = \xi' y - \xi(u+x) \quad \text{for } \lambda=1$$

are introduced.

The last notation gives convenient expressions for different observables when a series expansion in powers of $\xi = \alpha Z/2\rho$ is used in calculations of the beta transition probability. Because six matrix elements can contribute to the $2- \rightarrow 2+$ beta decay, a determination of the significance of each of these matrix elements cannot be done from the beta-gamma directional correlation only. The correlation, however, can be used as a convenient guide in the choice of approximations for the contributions of the different matrix elements. In the case of Rb^{86} decay, other observables which have been studied are the shape of the $2- \rightarrow 2+$ beta spectrum [5], the circular polarization of the 1.08 MeV gamma ray [6], and the branching ratio for the $2- \rightarrow 2+$ and the $2- \rightarrow 0+$ beta transition [7]. We shall attempt to find a consistent agreement with theory for these four observables.

The usual ξ -approximation, in which only the leading non-vanishing term for any observable is kept in the series expansion of $\xi = \alpha Z/2\rho$, cannot be used to describe the Rb^{86} decay. Earlier, Kotani [1d] has pointed out that the large value of $\varepsilon/(p^2/W) \geq 0.1$, obtained by Stevenson and Deutsch [3], indicated the failure of the ξ -approximation. Our value for $\varepsilon/(p^2/W)$ is even larger, as shown in Fig. 4.

The failure of the ξ -approximation is further supported by the fact that the shape of the beta spectrum [5] deviates from a linear Fermi-Kurie plot by about 10 per cent, whereas the ξ -approximation predicts an essentially linear F-K plot. Thus, more terms in the series expansion must be taken into account.

The first term in the expansion for the spectral shape, the β - γ correlation,

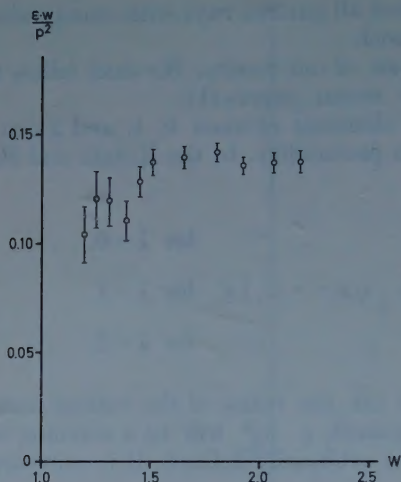


Fig. 4. The beta-gamma directional correlation coefficient, divided by the principle energy dependence P^2/W , as a function of energy.

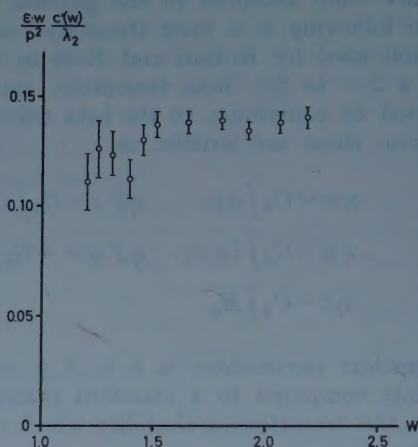


Fig. 5. Plot of $\epsilon \cdot \frac{W}{P^2} \cdot \frac{C'(W)}{\lambda_2}$ vs W where $C'(W)$ is taken from the work of Robinson and Langer [5].

etc., may be small either because of cancellations among the nuclear matrix elements appearing in the terms or because of the effects of selection rules which reduce the magnitudes of the matrix elements themselves. In either case, higher order terms in the expansion may become significant.

In the shell model, j -forbiddenness may be an important selection rule, whereas in the region of deformed nuclei the effects of the K selection rule must be considered. In the case of $_{37}\text{Rb}^{86}$, the last proton and neutron both lie in the 28–50 shell, which has the single particle states $f_{5/2}$, $p_{3/2}$, $p_{1/2}$, and $g_{9/2}$. Of these only the $g_{9/2}$ state has even parity. The fact that the Rb^{86} beta-decay involves a parity change means that the number of nucleons occupying the $g_{9/2}$ state must change by one unit. This implies $\Delta j \geq 2$, hence the main contribution to the Rb^{86} beta transition might be expected to arise from the B_{ij} matrix element, with $\lambda = 2$. The experimental evidence, however, indicates that a considerable contribution to the transition probability comes from matrix elements of lower rank. That is, the B_{ij} matrix element alone would produce a negative anisotropy in the β - γ correlation, and the spectrum would have the usual “unique” shape, whereas actually the anisotropy is positive and the spectrum does not have the “unique” shape.

Because of the proximity of Rb^{86} to the 50-neutron shell, it is not expected that a description of the states in terms of the K -quantum number will be valid, hence K -forbiddenness should not contribute significantly to the reduction of matrix elements in Rb^{86} decay.

Since it is possible for six matrix elements to contribute in $2- \rightarrow 2+$ beta decay, the analysis of the data is facilitated by making initial approximations about the expected magnitudes of the various matrix elements (see reference [1d] for examples). The “modified B_{ij} approximation”, proposed by Matumoto, Morita, and Yamada [1], is convenient for use in looking for selection rule effects. In

this approximation, $x=u=w=0$ and z , Y , and $V \neq 0$, compatible with a leading B_{if} term and non-zero relativistic matrix elements. It may be seen from the theoretical expressions for the spectrum shape factor (equations 25 and 26 in reference [1d]) that the "modified B_{if} approximation" predicts a minimum in the measured shape at around $E_\beta = 100$ keV, whereas the observed minimum occurs at about $E_\beta = 400$ keV. However, we shall show that this approximation provides a reasonable consistency for all the other observables thus far measured in the Rb^{86} beta decay.

In the B_{if} approximation, the expression for the β - γ directional correlation [1 d] is

$$\varepsilon = \frac{p^2}{W} \cdot \frac{R_3 + e W}{C'(W)},$$

where $C'(W)$ = the energy dependent shape factor

$$R_3 k = \lambda_2 [Y - (8/3)^{1/2} V] (1/56)^{1/2} \quad (\text{eq. 27 a, ref. [1 d]})$$

$$ek = -(\lambda_1/112) \quad (\text{eq. 27 b, ref. [1 d]})$$

$$k = V^2 + Y^2 + \frac{1}{12}(W_0^2 - \lambda_1) \quad (\text{eq. 27 c, ref. [1 d]}) \text{ and}$$

λ_1 and λ_2 are Coulomb correction factors which are nearly unity.

A plot of $\varepsilon(W/p^2) \cdot C'(W)$ versus W is thus a convenient way to represent the data and to determine R_3 and e . In Fig. 5 we show a plot of this function, for our data. For $C'(W)$ we have used the empirical equation $C'(W) = 1 + b/W$ with $4.4 < b < 0.6$, as given by Robinson and Langer [5]. From Fig. 5, we find $R_3 \approx 0.13$ and $e \approx 0$. Thus, from the above equations and from our results we have

$$R_3 = \frac{0.133 [Y - 1.63 V]}{V^2 + Y^2 + 0.40} \approx 0.13.$$

We may also use the known branching ratio in the Rb^{86} decay (8.7 per cent beta decay to the $2+$ state in Sr^{86} , 91.3 per cent to the $0+$ ground state) to estimate the magnitudes of V and Y , since

$$\frac{\text{ft}(2- \text{ to } 0+)}{\text{ft}(2- \text{ to } 2+)} = \frac{V^2 + Y^2 + Z^2}{Z^2} = V^2 + Y^2 + 1.$$

The value (8.7 ± 0.4) per cent is an average of two results (8.9 ± 0.5) per cent [7a] and (8.5 ± 0.5) per cent [7b] obtained from measurements of the gamma rays and total beta rays. The corresponding value obtained from beta spectra analysis (16 per cent) has a much greater uncertainty [10] and was not used. From the value 8.7 per cent for the branching of the $2-$ to $2+$ transition, we obtain $V^2 + Y^2 = 2.15$.

From these equations in V and Y we obtain two sets of solutions.

$$\text{Set I: } V \approx -1.45, Y \approx 0.1;$$

$$\text{Set II: } V \approx -0.7, Y \approx +1.3.$$

Table 1. Energy dependence of the circular polarization coefficient derived from solutions I and II.

W	ω	
	Set I	Set II
1.5	+0.022	+0.15
1.8	+0.006	+0.09
2.0	+0.002	+0.05

It would be desirable now to make use of our knowledge of other observables in order to choose between these two solutions. The spectrum shape factor $C'(W) = 1 + aW + b/W + cW^2$ contains combinations of matrix elements in the coefficients a , b , and c . However, because the shape of the Rb⁸⁶ spectrum was found by Robinson and Langer [5] to differ little from linearity over the energy region investigated, it was not possible to deduce definitive values of a , b , and c . Therefore, the shape factor cannot be used to distinguish between solutions I and II. Both solutions are consistent with values of a , b , and c which reproduce the measured shape and also indicate values of x , u , and w small with respect to unity.

Our deduced values of V and Y clearly indicate the breakdown of the ordinary ξ -approximation, as V and Y are of the same magnitudes as Z , whereas a necessary condition for the ξ -approximation is $V; Y \gg Z$.

Another method which in principle should enable us to select between the two solutions is the analysis of the measured beta-circularly polarized gamma correlation coefficient ω (see equations A18 to A25 in reference [1d]); the "modified B_{ij} approximation" predicts an energy dependence for ω , hence a knowledge of this dependence can be useful for the determination of the matrix elements.

A value of ω is known from the work of Boehm [6] ($\omega = 0.08 \pm 0.09$ at $\theta = 148^\circ$) but this is an integral value, as all beta energies above ~ 200 keV were accepted. We have calculated from our solutions I and II the values of ω at $W = 1.5$, 1.8, and 2.0 predicted by the "modified B_{ij} approximation". Unfortunately, the uncertainty in the measured circular polarization coefficient prevents a choice between the two sets. The calculations are not very reliable anyway, because several terms containing x , u , and w were neglected. It is possible that such terms, even though small, could be important if they all add in one direction and a cancellation between the main terms occurs.

It is interesting to note that solution I, which is essentially consistent with all measured variables, is in strong contradiction to K -forbiddenness hypotheses, which would demand $|Y| > |V|$. This condition, on the other hand, is fulfilled by solution II. J -forbiddenness allows no prediction to be made about the relative magnitudes of V and Y .

Conclusion

The beta-energy dependence of the beta-gamma directional correlation in Rb⁸⁶ has been measured. The magnitude of the correlation is about twice as large as the earlier result of Stevenson and Deutsch [3]. In spite of the improved knowl-

edge of the beta-gamma correlation,¹ our present knowledge of all observables in this 2^- to 2^+ beta transition is not sufficient to allow definite conclusions to be made about the relative magnitudes of the matrix elements which contribute to the transition. It has been shown, however, that all measured observables are essentially consistent with the shell model picture of Rb^{86} which predicts the beta transition to be J -forbidden. The systematic occurrence of high ft values for 2^- to 2^+ beta transitions in the region $28 < Z$, $N < 50$ and $50 < Z$, $N < 82$ further implies that cancellations among the matrix elements are unlikely to be responsible for the dominance of matrix elements of high rank.

ACKNOWLEDGEMENTS

The authors wish to thank Professors T. Kotani and M. Ross for many helpful discussions of the theoretical aspects of this work. We wish to express our appreciation to Professor Kai Siegbahn for placing the facilities of the Laboratory at our disposal. Two of us (J. H. Hamilton and J. M. Hollander) are grateful for the gracious hospitality extended us by Prof. Siegbahn and many others at the Institute of Physics. It is with gratitude that one of us (J. H. Hamilton) acknowledges a National Science Foundation Post-doctoral Fellowship and another (J. M. Hollander) a John Simon Guggenheim Fellowship, which made these studies possible.

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¹ This correlation has also been measured recently by H. Fishbeck and R. G. Wilkinson [12], with results in essential agreement with those reported here.

